FUNDAMENTALS OF CORPORATE CURRENCY EXPOSURE

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Thanks to Uluc Aysun, Stefano Mazzotta, and Joachim Vogt for helpful comments.

Abstract: I model the relation between corporate currency exposure and fundamental variables like demand elasticities and operating cost structure. The currency location of a firm’s operating costs may be in the home currency, the foreign currency, or partially in each. I start with a single-firm setting and extend the results to a competitive setting. The model should help managers better understand the determinants of currency exposure and thus better perform the important tasks of strategic planning and managing enterprise risk.

Keywords: Currency exposure; demand elasticity; operating costs; competition; exchange rate

JEL Classifications: G13, G32
I present a simple model of a firm’s currency exposure in terms of basic variables, including demand elasticities and operating cost characteristics. The model should help managers better understand the determinants of currency exposure and thus better perform the important tasks of strategic planning and managing enterprise risk.

I measure currency exposure as the exchange rate elasticity of operating profit, as in Bodnar, Dumas, and Marston (2002) (BDM). However, my model is more basic, and I consider a number of variables that BDM do not.

Ignoring the impact of exchange rate changes on aggregate income, I first show the following basic results for an exporter in a single-firm environment: 1) If all variable operating costs are based in the home currency, the currency exposure of gross operating profits (before fixed operating costs) has a simple direct relation with the price elasticity of demand and a simple inverse relation with the variable operating cost convexity; 2) If all variable operating costs are based in the foreign currency, the currency exposure of gross operating profits is 1, regardless of the price elasticity and variable operating cost convexity; 3) If variable operating costs are based partly in the home currency and partly in the foreign currency, the exposure is a linear combination of the exposures of the two extreme cases, weighting by the proportions of foreign currency and home currency variable operating costs.

I next model shocks to the location of the demand curve due to macroeconomic foreign exchange (FX) exposure, i.e., the exchange rate elasticity of aggregate income (or wealth). In
this case, a firm’s currency exposure also depends in a simple way on this elasticity and on the
product’s income elasticity of demand. Moreover, the overall impact of price elasticity on a
firm’s currency exposure depends on these two variables. It is theoretically possible for a
higher price elasticity to imply a lower currency exposure, and for an exporter’s currency
exposure to be less than 1, or even negative.

I extend the analysis to a homogeneous-product Cournot competition, showing that a
firm’s currency exposure is a simple function of the foreign currency proportion of variable
operating costs of the industry as a whole, and of the firm relative to the industry. If a firm has
the same foreign currency proportion of variable operating costs as the overall industry, the
currency exposure of gross operating profit is the same as in a single-firm setting, regardless of
market share or industry concentration. If a firm has a higher foreign currency proportion of
variable operating costs than the overall industry, currency exposure may be inversely related
to price elasticity.

I use Booth’s (1991) flexible production approach, so that a firm may adjust output (as
well as price) when demand conditions change unexpectedly. I also assume isoelastic demand,
consistent with the log-linear approach that economists often use to estimate demand
elasticities using empirical data. Isoelastic demand is no more restrictive than any other type of
demand assumption, such as linear demand and constant elasticity of substitution (CES)
demand. My assumptions yield a relatively simple model of currency exposure that should be a
useful approximation of models derived under alternate assumptions.

I. Relation to the Literature

Following the pioneering work of Adler and Dumas (1984), Marston (2001) defines
currency exposure as the derivative of operating profit with respect to the exchange rate,
instead of the elasticity of operating profit with respect to the exchange rate. This definition implies that an exporting monopolist’s currency exposure is equal to the net revenues in the foreign currency, obscuring the roles of price elasticity and variable operating cost convexity. In fact, since higher price elasticity implies lower mark-up and thus lower net revenues, higher product price elasticity implies lower currency exposure with Marston’s definition.

More complex demand environments and oligopoly structures are found in the models of Dornbush (1987), Marston (2001), and Bodnar, Dumas, and Marston (2002). For example, Bodnar, Dumas, and Marston (BDM) assume a constant elasticity of (product) substitution (CES) consumer utility function and that an exporter’s product competes imperfectly against the product of a foreign firm. In the BDM model, price elasticity is not constant, so the relation between price elasticity and currency exposure is complex, and the competing firms’ currency exposures are related to market shares.

The BDM model allows the degree of product substitutability to range from nearly 0 to nearly 100%, but is degenerate at the extreme points. So the BDM model cannot yield as special cases a single-firm scenario or a homogeneous-product competition. I access these basic settings with isoelastic demand and get simple relations and straightforward insights. Moreover, my model has variables that the BDM model does not, since BDM assume 1) linear variable operating costs; 2) no macroeconomic FX exposure; and 3) a competitor’s foreign currency proportion of operating costs of 100%.

Campa and Goldberg (1999) and Allayannis and Ihrig (2001) find empirically that currency exposures are greater for firms with lower mark-ups. These results are consistent with both my model and that of BDM. In my model, an exporter with a higher price elasticity has a lower mark-up and higher currency exposure. In the BDM model, an exporter whose product has higher substitutability has a lower mark-up and a higher currency exposure.
Entorf, Moebert, and Sonderhof (2007) show empirically that, other things equal, a net exporting nation tends to have higher income when its currency falls, and a net importing nation tends to have higher income when its currency rises. They also show that the size and direction of FX exposure of a nation’s income depends on the size and sign of the current account balance. Since a nation’s current account balance depends on the exchange rate, the FX exposure of income is likely to be time-varying, as found empirically by Chaieb and Mazzotta (2008). In turn, corporate currency exposure is likely to be time-varying, as found empirically by Doukas, Hall, and Lang (2003). These empirical results are the basis for my attempt to model the impact of macroeconomic FX exposure.

As I said, I find that currency exposure is greater when variable operating costs are less convex in output. If larger firms have less convex variable operating costs due to economies of scale of production, we would expect higher currency exposure in larger firms even after controlling for the extent of international operations, as was found empirically by He and Ng (1998), Bodnar and Wong (2003), and Doidge, Griffin, and Williamson (2006).

Muller and Verschoor (2006) present a comprehensive review of the current theory of corporate currency exposure, pointing out that “there is (no) real consensus concerning the relevant parameters influencing currency risk exposure.” I hope that my analysis offers accessible insights into the fundamental determinants of corporate currency exposure.

My purpose is to provide basic insights on currency risk. I include no empirical analysis for the following reasons: First, some excellent empirical research has already been published, as I reference. Second, I am not sure which of my theoretical results represent testable hypotheses and which are just logical insights from elementary microeconomics, so I leave it to empiricists to identify any research opportunities that the model provides.
II. The Basic Single-Firm Model

I assume initially an exporter in a single-firm environment. The exporter faces a downward-sloping, isoelastic, inverse demand curve in the foreign country:

\[ P_f = aQ^{-\frac{1}{\varepsilon}} \]  \hspace{1cm} (1)

where \( P_f \) is the product’s price in the foreign currency, \( Q \) is the firm’s output, and \( \varepsilon \) is the absolute value of the product’s constant price elasticity of demand. In a single-firm setting \( \varepsilon \) must be greater than 1, but this is not necessary in a competitive setting as long as the firm-level price elasticities are greater than 1. The location of the demand curve is the parameter \( a \), which we assume for now is not affected by exchange rate changes.

The exporter’s variable operating costs, \( V \), are given by the function in equation (2):

\[ V = vQ^c \]  \hspace{1cm} (2)

where \( v \) is the coefficient of all of the firm’s variable operating costs, measured in the home currency, and \( c \) reflects the convexity of the variable operating cost function. I assume also that \( v \) and \( c \) are positive, constant, non-stochastic, and independent of the exchange rate.

The convexity of the variable operating cost function, \( c \), varies from firm to firm and depends on the on the costs and productivities of factor inputs, e.g., labor, energy, and materials. All else the same, a firm operating at or near full capacity is likely to have a marginal operating cost function that increases in output (\( c > 1 \)).

\[ 1 \] A survey of executives in various industries by Blinder et al. (1998) reveals a range of managerial opinion about marginal operating costs. Bairam (1998) estimates \( c \) for 58 large US companies using data from 1975 through 1992. Bairam’s estimates suggest that some companies have marginal operating costs
An exporter with some portion of operating costs based in the foreign currency is doing operational hedging. Examples include assembling the final product in the foreign country or sourcing inputs whose price is fixed in the foreign currency. For tractability, I assume the variable operating costs based in the home currency have the same $c$ as those based in the foreign currency.

Let $X$ denote the exchange rate in home currency per foreign currency, $v_h$ the coefficient for variable operating costs based in the home currency, and $v_f$ the coefficient for variable operating costs based in the foreign currency. Thus, $v = v_h + Xv_f$. The foreign currency proportion of the variable operating costs, denoted $\gamma$, is shown in equation (3):

$$\gamma = \frac{Xv_f}{v_h + Xv_f} \quad (3)$$

The firm’s gross operating profit (in home currency), defined as operating profit before fixed operating costs and denoted $\pi_G$, is equal to revenues, $R$, minus variable operating costs, $V$, as shown in equation (4):

$$\pi_G = R - V \quad (4)$$

From the perspective of the home currency, the exposure of the revenues to the foreign currency is denoted $\zeta_R$. As in Bodnar, Dumas, and Marston (2002), $\zeta_R$ is an elasticity measured by the derivative $d(\ln R)/d(\ln X)$. I show in the appendix that $\zeta_R = \delta + \gamma(1 - \delta)$, which may be rearranged to the expression in equation (5):

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that increase with output ($c > 1$), some that decline ($c < 1$), and some that are very close to constant ($c = 1$). The highest of Bairam’s $c$ estimates is 1.56 for PepsiCo. The lowest is 0.77 for Deere. In general, Bairam finds that oil, petrochemical, chemical, and soft drink companies had $c$ estimates greater than 1 (increasing marginal operating cost), while manufacturing firms tended to have $c$ estimates less than 1 (declining marginal operating cost).
\[ \xi_R = \gamma + (1 - \gamma)\delta \]  

(5)

where \( \delta \) is defined in equation (6):

\[ \delta = c/(c - (1 - \varepsilon^{-1})) \]  

(6)

Equation (5) says that revenue exposure is a weighted average of 1 and \( \delta \), which are the revenue exposures that would hold under the extreme cases of variable operating costs entirely in the foreign currency (\( \gamma = 1 \)) and entirely in the home currency (\( \gamma = 0 \)). The weights are \( \gamma \) and \( (1 - \gamma) \), which are the proportions of variable operating costs in the foreign currency and home currency, respectively.

Unless all of the variable operating costs are based in the foreign currency, \( \delta \) is the driver of the firm’s revenue exposure. From equation (6), we see that \( \delta \) depends on \( \varepsilon \) and \( c \). The derivative of \( \delta \) with respect to \( \varepsilon \), \( d\delta/d\varepsilon \), is \( \varepsilon^{-2}c/[c - (1 - \varepsilon^{-1})]^2 \), which is always positive. So unless \( \gamma = 1 \), higher price elasticity implies higher currency exposure, all else the same. The derivative of \( \delta \) with respect to \( c \), \( d\delta/dc \), is \( -(1 - \varepsilon^{-1})/[c - (1 - \varepsilon^{-1})]^2 \), which is always negative. So unless \( \gamma = 1 \), higher variable operating cost convexity implies lower currency exposure, all else the same.

If the variable operating costs are based entirely in the foreign currency (\( \gamma = 1 \)), the firm’s revenue exposure is 1, regardless of \( \varepsilon \) and \( c \). In this case, the firm makes no changes to output or product price (in the foreign currency) when the exchange rate changes; the only currency exposure of the revenue is the conversion of the constant foreign currency revenue into the home currency.
The firm’s variable operating costs (measured in the home currency), $V$, also change when the exchange rate changes. If $\gamma > 0$, $V$ changes because of the conversion of the variable operating costs that are based in the foreign currency. If $1 > \gamma$, $V$ also changes because of the change in the optimal output. The overall elasticity of $V$ to an exchange rate change is the currency exposure of the firm’s variable operating costs. We denote this exposure as $\xi_V$ and measure it by the derivative $d(\ln V)/d(\ln X)$. I show in the appendix that $\xi_V = \delta + \gamma(1 - \delta)$, which may be rearranged to the expression in equation (7):

$$
\xi_V = \gamma + (1 - \gamma)\delta 
$$

(7)

So we see in equation (7) that the currency exposure of the firm’s variable operating costs is exactly the same as the currency exposure of the firm’s revenues.

To find the currency exposure of the firm’s gross operating profits, denoted $\xi_G$, we’ll use the Bodnar-Marston (2002) model shown in equation (8):

$$
\xi_G = \xi_R(R/\pi_G) - \xi_I(V/\pi_G) 
$$

(8)

Since $\pi_G = R - V$ and $\xi_R = \xi_V = \gamma + (1 - \gamma)\delta$, we can use equation (8) to see that the currency exposure of the firm’s gross operating profits is the same as both the revenue exposure and the variable operating cost exposure, as shown in equation (9):

$$
\xi_G = \gamma + (1 - \gamma)\delta 
$$

(9)

FX pass-through measures the elasticity of the product price (in the foreign currency) to changes in the exchange rate, measured in terms of the foreign currency price of the exporter’s home currency, $1/X$. Formally, FX pass-through is measured by the derivative
\[ \frac{d(\ln P)}{d(\ln[1/X])} \]. Taking this derivative using equation (A3), we find that the FX pass-through is equal to \((1 - \gamma)\eta\), where \(\eta = \delta(ce)\). If the variable operating costs are based entirely in the home currency \((\gamma = 0)\), the FX pass-through is equal to \(\eta\). If the variable operating costs are based entirely in the foreign currency \((\gamma = 1)\), the FX pass-through is equal to 0.

Table I contains a numerical example.

### TABLE I: SINGLE-FIRM EXAMPLE

<table>
<thead>
<tr>
<th>ASSUMPTIONS</th>
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<tr>
<td>(X)</td>
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<td></td>
<td>FX pass-through</td>
<td>(v_h + Xv_f)</td>
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\(\eta = \delta(ce)\)

9
III. Economics of Exporter’s Currency Exposure

Next we analyze currency exposure if the exporter’s variable operating costs are based entirely in the home currency ($\gamma = 0$). In this case as we said, the currency exposure is equal to $\delta$ and the FX pass-through is equal to $\eta$.

If $c = 1$, the variable operating costs are linear in output, and marginal cost is constant. In this case, $\delta = \varepsilon$ and $\eta = 1$. That is, the currency exposure is equal to the price elasticity, and the FX pass-through is 100%. If marginal operating costs are constant, a change in the exchange rate results in a change in the optimal output quantity but not in the product price as viewed in the exporter’s home currency, so the FX pass-through is 100%. The elasticity of optimal output quantity to exchange rate changes is equal to the price elasticity. With no change in product price as viewed in the exporter’s home currency, the elasticity of revenues (in the home currency) to exchange rate changes is also equal to the price elasticity of the exporter’s product.

If $c > 1$, the marginal operating cost increases as output increases. In this case, $\delta < \varepsilon$, and $\eta < 1$. That is, the currency exposure is less than the price elasticity, and the FX pass-through is less than 100%. We see this case in the numerical example in Table I, where $\varepsilon = 2$ and $c = 1.25$, implying that $\delta$ is 1.67 and $\eta$ is 0.67.

With increasing marginal operating costs, a given change in the exchange rate means that the exporter would make a smaller change to the optimal output quantity than if marginal cost were constant, but also would adjust price as viewed in the home currency. Then FX pass-through would be less than 100%.
All else the same, a firm with a more steeply rising marginal operating cost function will make smaller adjustments in both optimal output quantity and product price in the foreign currency. Thus, revenue (in home currency) has a lower elasticity to a given exchange rate change, and the impact of the price elasticity is smaller. As the marginal operating cost function approaches verticality in the limit, the exporter would make no change in output or the product price in the foreign currency when the exchange rate changes, and the revenue exposure converges to the one-for-one conversion exposure of the foreign revenues, regardless of the price elasticity.

The effects of price elasticity on price and output decisions work in the opposite way for firms with declining marginal operating rather than increasing. The firm’s marginal operating cost declines as output increases if \( c < 1 \). In this case, \( \delta > \varepsilon \) and \( \eta > 1 \). That is, the currency exposure is higher than the price elasticity, and the FX pass-through is greater than 100%.

All else the same, a firm with a higher price elasticity will make greater changes to both optimal output and price (in the foreign currency) in the face of declining marginal operating costs. Thus, revenues in the home currency are more exposed to changes in the exchange rate, and the FX pass-through is higher than 100%. Note that \( c \) must be greater than \( 1 - \varepsilon^{-1} \), or the downward slope of the marginal operating cost function would be too steep for a solution with marginal revenue.

The FX pass-through results in this section are specific examples of Knetter’s (1994) general model. Hau (1999) and Marston (2001) also point out the 100% FX pass-through results when variable operating costs are linear and entirely in the home currency.
IV. Macroeconomic FX Exposure

The analysis so far assumes the demand curve does not shift when the exchange rate changes. The location of a product’s demand curve, however, typically depends on the aggregate income (or wealth) of the buyers. And a nation’s income may depend on the value of the nation’s currency relative to other currencies, as shown empirically by Entorf, Moebert, and Sonderhof (2007).

Consider an exporter to a country whose aggregate income, \( Y \), has FX exposure \( \mu \) to changes in \( X \). That is, \( Y = mX^\mu \), where \( \mu = d(\ln Y)/d(\ln X) \), and \( m \) captures all other determinants of \( Y \). If \( \mu > 0 \), the country’s income tends to rise as its currency appreciates (\( X \) is higher), characteristic of a net importing nation with a current account deficit. If \( \mu < 0 \), the country’s income tends to fall as its currency appreciates, characteristic of a net exporting nation with a current account surplus. The FX exposure of a country’s aggregate income is likely to be time-varying, since the current account balance is partly a function of the currency value.

We now express the demand function for the exporter’s product as \( Q = a'Y^bP_f^{\varepsilon} \), where \( b \) is the product’s income elasticity of demand. Most products are normal goods, where demand increases as aggregate income increases; in this case, income elasticity is positive, i.e., \( b > 0 \). More essential goods have relatively low (but positive) income elasticity, e.g., \( 1 > b > 0 \). For the relatively rare case of an inferior good, income elasticity is negative, i.e., \( b < 0 \).

The currency exposure of an exporter’s gross operating profits is derived in the appendix and shown in equation (10):

\[
\xi_G = \gamma + (1 - \gamma + \mu b \varepsilon^{-1}) \delta \tag{10}
\]
We see the additional impact of FX macroeconomic exposure on a firm’s currency exposure in the $\mu b \varepsilon^{-1}$ term in the parentheses in equation (10). Assume a firm that exports a normal good ($b > 0$). If the firm exports to a country that is a net importer from the exporter’s country ($\mu > 0$), the macroeconomic FX effect increases the currency exposure. The exporter’s $\xi_G$ may be greater than 1 even if all of the exporter’s operating costs are based in the foreign country ($\gamma = 1$). If a firm exports to a country that is a net exporter to the exporter’s country ($\mu < 0$), the macroeconomic FX effect reduces the currency exposure. The exporter’s $\xi_G$ may be less than 1 and even less than zero. Equation (9) is a special case of equation (10) when $\mu = 0$, i.e., when the location of the demand curve is not affected by the exchange rate.

As we can see, the size of the macroeconomic FX effect on a firm’s currency exposure depends on the product’s price elasticity of demand, $\varepsilon$, in conjunction with the FX exposure of the export market’s aggregate income, $\mu$, and the product’s income elasticity of demand, $b$. Because of this additional role for $\varepsilon$, currency exposure may be inversely related to price elasticity if enough of the firm’s variable operating costs are based in the foreign currency (high $\gamma$).

For some numerical examples, assume the product’s income elasticity of demand is $b = 2$. As in the example in Table I, where $\mu$ is implicitly 0 and $\xi_G = 1.40$, we assume $\varepsilon = 2$, $c = 1.25$, $\delta = 1.67$, and $\gamma = 0.40$. By equation (10), if $\mu = 0.80$, the currency exposure is $\xi_G = 0.40 + [1 – 0.40 + 0.80(2)(0.50)]1.67 = 2.73$; if $\mu = –0.60$, the currency exposure is $\xi_G = 0.40 + [1 – 0.40 – 0.80(2)(0.50)]1.67 = 0.40$.

For an example where currency exposure is inversely related to price elasticity, assume a relatively high proportion of foreign currency variable operating costs, $\gamma = 0.75$, and hold all else the same as above, including $\mu = 0.80$. By equation (10), $\xi_G = 0.75 + [1 – 0.75 +
0.80(2)(0.50)[1.67] = 2.50. Now assume a lower price elasticity, \( \varepsilon = 1.50 \), so \( \delta = 1.36 \), all else the same. Using equation (10), \( \xi_G = 0.75 + [1 - 0.75 + 0.80(2)(0.67)]1.36 = 2.54 \), higher than the 2.50 we just found for \( \varepsilon = 2 \).

V. Operating Leverage and Currency Exposure

A firm’s degree of operating leverage refers to its relative use of fixed operating cost inputs instead of variable operating cost inputs. Let us define the exporter’s net operating profit (in home currency) as \( O = \pi_G - F \), where \( F \) denote the total fixed operating costs measured in the home currency: \( F = F_h + XF_f \), where \( F_h \) denotes the fixed operating costs based in the home currency, and \( F_f \) denotes the fixed operating costs based in the foreign currency. From the home currency perspective, the exposure of \( F_h \) to changes in \( X \) is 0, while the exposure of \( XF_f \) to changes in \( X \) is 1.

So we can apply the Bodnar-Marston model in equation (8), with the optimal \( \pi_G \) playing the role of \( R \), and the optimal \( O \) playing the role of \( \pi_G \). We get that the currency exposure of the firm’s optimal net operating profit, which we’ll call the firm’s FX operating exposure and denote \( \xi_O \), is equal to \( \xi_O = \xi_G(\pi_G/O) - XF_f/O \). We can rearrange the last expression to obtain equation (11):

\[
\xi_O = \xi_G(1 + F/O) - XF_f/O \tag{11}
\]

In equation (11), the \( 1 + F/O \) term measures the degree operating leverage. The \( XF_f/O \) term is an adjustment for the operational hedging aspect of the fixed operating costs that are based in the foreign currency. Typically, the higher the degree of operating leverage, other
things equal, the higher the FX operating exposure, but less so to the extent that any of the fixed operating costs are based in the foreign currency and thus perform operational hedging.

There is one special case where the degree of operating leverage has no impact on FX operating exposure: if all variable and fixed operating costs are based entirely in the foreign currency, and if there is no macroeconomic FX effect on the location of the demand curve. In this case, the firm’s FX operating exposure is 1, regardless of the structure of fixed and variable operating costs. You can see this from equation (11), noting that \( \xi_G = 1 \). The reason is that the firm makes no output adjustments in response to exchange rate changes. If output is stable, there is no distinction between fixed and variable operating costs, and so the degree of operating leverage does not affect the volatility of net operating profit.

VI. Domestic “Importer”

We can adapt the exporter analysis to a firm that sells a finished product in its domestic market, by simply reversing the currency perspective. For simplicity, we’ll call the domestic firm an importer if any of its operating costs are based in an overseas currency, directly or indirectly. The domestic importer may even source its finished product from overseas.

We can adapt the exporter analysis because a firm that is an exporter from one currency perspective is an importer from the other currency perspective. To help keep things straight, we’ll say that the importer’s domestic currency is what we have called the foreign currency for the exporter, and the overseas currency for the importer is what we have called the exporter’s home currency.

The importer’s exposure to the overseas currency is the simple Adler-Jorion (1992) transformation of the exporter’s exposure to the foreign currency, as in equation (12):
By sourcing inputs whose cost is based in the overseas currency, the importer faces an obvious negative exposure to the overseas currency. In addition, the importer will adjust the product price and output if variable operating costs change from the perspective of the importer’s domestic currency. These adjustments imply that revenues change with FX rates, i.e., that the firm has revenue exposure to the overseas currency. Thus, the price elasticity of demand and shape of the marginal operating cost function affect the revenue exposure.

From our analysis of an exporter whose variable operating costs are entirely in the foreign currency, we can use equation (12) for a firm that imports a finished product from overseas for distribution in the domestic market. For example, with a constant variable operating cost per unit based entirely in an overseas currency, the currency exposure of the firm’s revenues is equal to one minus the product’s price elasticity of demand.

**VII. Cournot Competition**

I now show that if competing firms in a homogeneous-product Cournot industry have the same $c$ and the same currency proportions of variable operating costs, the firms have the same currency exposure of gross operating profit, even if the firms have different market shares. And the currency exposure is the same that any of the companies would have in a single-firm setting. If the firms have different currency proportions of variable operating costs, an additional factor affecting a firm’s currency exposure is the industry’s overall foreign currency proportion of variable operating costs. For simplicity, I assume now that all firms have linear variable operating costs, i.e., $c = 1$, but the same general ideas apply if the firms have the same $c \neq 1$. For focus, I ignore macroeconomic FX exposure and operating leverage in this section.
If the competing firms have different \( v_i \)'s, the firms will have different market shares. Let \( v \) denote \( \Sigma v_i; \ v_f \) denote \( \Sigma v_{if}; \ \gamma_i \) denote \( X_{v_{if}}/v_i \), firm \( i \)'s foreign currency proportion of variable operating costs; and \( \gamma \) denote \( X_{v}/v \), the overall industry’s foreign currency proportion of variable operating costs. Let \( G_i \) denote firm \( i \)'s gross operating profit margin, \( \pi_{Gi}/R_i \). By some tedious steps in the appendix, we get firm \( i \)'s revenue exposure, \( \xi_{R_i} \), as given in equation (13):

\[
\xi_{R_i} = \gamma' + (1 - \gamma)\epsilon
\]  

(13)

where

\[
\gamma_i' = [\gamma - \gamma(1 - G_i)]/G_i
\]

(14)

Equation (13) resembles equation (5) for the single-firm case, except we find \( \gamma' \) instead of \( \gamma \) as the first term, and we use \( \epsilon \) in place of \( \delta \) since \( c = 1 \). As we see in equation (14), \( \gamma' \) depends on the firm’s foreign currency variable operating costs, \( \gamma_i \), relative to the overall industry’s, \( \gamma \), and on \( G_i \), the firm’s gross operating profit margin, which is also equal to \( 1 - (v_i/v)(n - \epsilon^{-1}) \).

The derivative of \( \xi_{R_i} \) with respect to \( \epsilon \) is equal to \( [(\gamma - \gamma)(v_i/v)]/(\epsilon G_i)^2 + (1 - \gamma) \). This derivative is positive if firm \( i \)'s foreign currency proportion of variable operating costs is less than the overall industry’s, i.e., if \( \gamma > \gamma_i \). If \( \gamma > \gamma_f \), the derivative may be negative if firm \( i \) has a high enough \( \gamma_i \). In the simple case of equal market shares (where \( n = v/v_i \)), the derivative simplifies to \( (\gamma - \gamma)(v_f/v) + (1 - \gamma) \), which is negative if \( \gamma - \gamma > (1 - \gamma)(v_f/v) \).

By steps shown in the appendix, we get firm \( i \)'s variable operating cost exposure, as given in equation (15):
Equation (15) resembles equation (7) for the single-firm case, except that $\gamma_i'$ replaces $\gamma$ in the first term, and we have the additional term, $\gamma_i - \gamma$, reflecting the difference between the firm’s foreign currency proportion of variable operating costs and the industry’s. Again, we use $\varepsilon$ in equation (15) instead of $\delta$, since $c = 1$.

The $\gamma_i'$ variable defined in equation (14) depends on how firm $i$’s foreign currency proportion of variable operating costs compares to the overall industry’s. If $\gamma_i = \gamma$, i.e., if firm $i$’s foreign currency proportion of variable operating costs is equal to the overall industry’s, equation (14) tells us that $\gamma_i'$ is equal to $\gamma$. In this case equations (13) and (15) simplify to their versions for the single firm case, $\xi_{Ri} = \gamma + (1 - \gamma)\varepsilon$, and $\xi_{Vi} = \gamma + (1 - \gamma)\varepsilon$. So $\xi_{Gi} = \gamma + (1 - \gamma)\varepsilon$.

This finding implies that if all the firms have the same currency proportion of variable operating costs, they will all have the same currency exposure of gross operating profits, regardless of industry concentration or whether the firms have different $v_i$’s and thus different market shares.

If $\gamma_i \neq \gamma$, i.e., if firm $i$’s foreign currency proportion of variable operating costs is not equal to the overall industry’s, firm $i$’s variable operating cost exposure in equation (15) does not equal to its revenue exposure in equation (13). So we need to use equation (8) to find $\xi_{Gi}$, firm $i$’s currency exposure of gross operating profits, given $R_i/\pi_{Gi}$ and $V_i/\pi_{Gi}$. Note that $R_i/\pi_{Gi}$ is the reciprocal of firm $i$’s gross operating profit margin. And $V_i/\pi_{Gi} = R_i/\pi_{Gi} - 1$.

For numerical examples, assume a duopoly ($n = 2$) in an industry where the product’s demand elasticity, $\varepsilon$, is 2. Assume that Firm 1’s $v_1$ is 4 and Firm 2’s $v_2$ is 6, so Firm 1 has the higher market share because of its lower variable operating costs. The gross operating profit
margin for Firm 1 is $G_1 = 1 - \left(\frac{v_1}{v}\right)(n - \epsilon^{-1}) = 1 - 0.40(2 - 0.50) = 0.40$. The gross operating profit margin for Firm 2 is $G_2 = 1 - \left(\frac{v_2}{v}\right)(n - \epsilon^{-1}) = 1 - 0.60(2 - 0.50) = 0.10$.

First assume that both firms have the same $\gamma_i = 0.60$. So 60% of the overall industry’s variable operating costs are based in the foreign currency, i.e., $\gamma = 0.60$. Although they have different market shares, both firms have the same currency exposure of revenues and variable operating costs, and thus the same currency exposure of gross operating profits: $\xi_{RI} = \xi_{VI} = \xi_{GI} = 0.60 + (1 - 0.60)^2 = 1.40$.

Next assume instead that Firm 1’s $v_1$ of 4 is based entirely in the home currency and Firm 2’s $v_2$ of 6 is based entirely in the foreign currency. So $\gamma_1 = 0$ and $\gamma_2 = 1$. Assume $X = 1$, so again, 60% of the overall industry’s variable operating costs are based in the foreign currency, i.e., $\gamma = 0.60$.

Using equation (14), $\gamma_1' = \frac{[0.60 - 0(1 - 0.40)]}{0.40} = 1.50$ and $\gamma_2' = \frac{[0.60 - 1(1 - 0.10)]}{0.10} = -3$. Firm 1 has a high positive $\gamma_1'$ due to having a lower foreign currency proportion of variable operating than the overall industry. Firm 2 has a high negative $\gamma_2'$ due to having a higher foreign currency proportion of variable operating than the overall industry.

By equation (13), the revenue exposures of the two competitors are $\xi_{R1} = 1.50 + (1 - 0.60)2 = 2.30$, and $\xi_{R2} = -3 + (1 - 0.60)2 = -2.20$. By equation (15), the variable operating cost exposures of the two firms are $\xi_{V1} = 1.50 + 0 - 0.60 + (1 - 0.60)2 = 1.70$, and $\xi_{V2} = -3 + 1 - 0.60 + (1 - 0.60)2 = -1.80$. Using equation (8), $\xi_{G1} = 3.20$ and $\xi_{G2} = -5.80$.

Table II summarizes the example, based on the assumption that $a = 20$. 

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### TABLE II: COURNOT COMPETITION EXAMPLE

<table>
<thead>
<tr>
<th>ASSUMPTIONS</th>
<th>FIRM 1</th>
<th>FIRM 2</th>
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</thead>
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</tr>
<tr>
<td>$\varepsilon$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
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<td></td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>$v_{if}$</td>
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<td>6</td>
</tr>
<tr>
<td>$v_i$</td>
<td>4</td>
<td>6</td>
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</table>

### CALCULATIONS

<table>
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<th>CALCULATIONS</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>$v_1 + v_2$</td>
</tr>
<tr>
<td>$v_f$</td>
<td>$v_{1f} + v_{2f}$</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
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<tr>
<td>$\gamma_i$</td>
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</tr>
<tr>
<td>$\gamma_i'$</td>
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</tr>
<tr>
<td>$\xi_Ri$</td>
<td>2.30</td>
</tr>
<tr>
<td>$\xi_v$</td>
<td>1.70</td>
</tr>
<tr>
<td>$R_i/\pi_Gi$</td>
<td>2.5</td>
</tr>
<tr>
<td>$V_i/\pi_Gi$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\xi_Gi$</td>
<td>3.20</td>
</tr>
</tbody>
</table>

| $Q_i$         | 7.20     | 1.80   |
| $P$           | 6.67     |        |
| $R_i$         | 48       | 12     |
| $V_i$         | 28.80    | 10.80  |
| $\pi_Gi$      | 19.20    | 1.20   |
VIII. Summary and Conclusion

My research objective was to build a simple model to see how an exporter’s currency exposure relates to the price elasticity of demand. Ignoring the impact of exchange rate changes on the location of the demand curve, the model yields the following results in the single-firm setting:

1) There is no relation between currency exposure and price elasticity if all of an exporter’s variable operating costs are based in the foreign currency. If any of the exporter’s variable operating costs are based in the home currency, currency exposure is directly related to the products’ price elasticity, and the impact depends on the convexity of the variable operating cost function.

2) If the variable operating costs are partly based in the home currency and partly based in the foreign currency, the currency exposure is a linear combination of the currency exposures of the two extreme cases for variable operating costs: all in the home currency, or all in the foreign currency.

3) For an exporter whose variable operating cost per unit is constant and based entirely in the home currency, the currency exposure of gross operating profits (before fixed operating costs) is equal to the price elasticity. If the variable operating cost per unit increases with output, the exposure is lower than the price elasticity. If the variable operating cost per unit declines with output, the exposure exceeds the price elasticity.

If exchange rate changes affect aggregate income and thus the location of the product demand curve, additional factors that affect an exporter’s currency exposure are the FX
exposure of aggregate income and the product’s income elasticity of demand. These two variables affect the relation between currency exposure and price elasticity.

Finally, I analyze a Cournot competition where firms may differ in market shares and in the foreign currency proportion of variable operating costs. I show that a firm’s currency exposure depends on the foreign currency proportion of variable operating costs of the overall industry, and of the firm relative to the overall industry. Currency exposure does not depend on market share.

The model in this paper provides basic insights on how fundamental forces determine a firm’s currency exposure. Hopefully, the results also open opportunities for empirical researchers.
REFERENCES


APPENDIX

Single-Firm Setting

The exporter’s total revenue (in the home currency), given $X$, is equal to $R = XP_fQ = X(aQ^{-1/\delta})Q = XaQ^{(1-1/\delta)}$. Therefore, the firm’s marginal revenue (in the home currency) is equal to $Xa(1 - \varepsilon^{-1})Q^{-1/\varepsilon}$. To maximize operating profit, the exporter sets marginal revenue equal to marginal operating cost, $cvQ^{(\varepsilon - 1)}$. With some algebraic manipulation, we solve for the optimal output, $Q^*$, as shown in equation (A1):

$$ Q^* = (Xa)^{\delta / \varepsilon} \quad (A1) $$

where $\delta = c/[c - (1 - \varepsilon^{-1})]$, as shown in equation (6), and

$$ z = (1 - \varepsilon^{-1})/(cv) \quad (A2) $$

Substituting the optimal output of equation (A1) into equation (1), we get the optimal product price (in foreign currency) shown in equation (A3):

$$ P_f^* = a(Xaz)^{-\delta(c\varepsilon)} \quad (A3) $$

The exporter’s revenue (in home currency), given the optimal output and product price, is

$$ R^* = XP_f^*Q^* = Xa(Xaz)^{-\delta(c\varepsilon)}(Xaz)^{\delta c} $$

which simplifies to equation (A4):

$$ R^* = Xa(Xaz)^{(1 - 1/\delta)\varepsilon} \quad (A4) $$
To find the revenue exposure to the foreign currency, take the derivative $d(\ln R)/d(\ln X)$ using equation (A4), recalling that $z$ depends on $v = \nu_h + X\nu_f$. The result is $\xi_R = \delta + \gamma(1 - \delta)$.

Plugging the expression for $Q^*$ from equation (A1) into equation (2), we get the firm’s variable operating costs (measured in the home currency) at the optimal price/output strategy:

$$V^* = \nu(Xaz)^{-\delta} \quad (A5)$$

Take the derivative $d(\ln V)/d(\ln X)$ using equation (A5), recalling that $z$ depends on $v = \nu_h + X\nu_f$. This result is $\xi_V = \delta + \gamma(1 - \delta)$.

**Macroeconomic FX Exposure**

Substituting $mX^\mu$ for $Y$ in the demand function, $Q = a'Y^b P_f^{-\varepsilon}$, we have $Q = a'(mX^\mu)^b P_f^{-\varepsilon}$. Rearranging, we get the expression for the indirect demand curve, $P_f = [a'(mX^\mu)^b]^{1/\varepsilon} Q^{1/\varepsilon}$. By comparing this expression with equation (1), we see that $a$ is now equal to $[a'(mX^\mu)^b]^{1/\varepsilon}$. Plug this expression for $a$ into equation (A4), and find $d(\ln R)/d(\ln X)$, the exporter’s revenue exposure to the foreign currency. The result is $\delta(1 + \mu b \varepsilon^{-1}) + \gamma(1 - \delta)$, which rearranges to $\gamma + (1 - \gamma + \mu b \varepsilon^{-1})\delta$. Doing the same for $a$ in equation (A5), we find the same result for $d(\ln V)/d(\ln X)$, the exporter’s variable operating cost exposure to the foreign currency. Relying on earlier reasoning, the currency exposure of the exporter’s gross operating profits is also the same as the currency exposure of revenues, as shown in equation (10).
Cournot Competition

Firm $i$’s total revenue in the home currency of an exporter is $R_i = XP_iQ_i = XaQ_i^{1/\epsilon}Q_i$, where $Q = \sum Q_i$. Since $dQ/dQ_i = 1$, firm $i$’s marginal revenue is $XaQ_i^{1/\epsilon} - \epsilon^{-1}XaQ_i^{(1 - 1/\epsilon)}Q_i$. Given $c = 1$, firm $i$’s marginal cost is $v_i$, where $v_i = v_{ih} + Xv_{if}$. To maximize profit, firm $i$ sets marginal revenue equal to marginal cost, $v_i$. Sum the $MR_i = v_i$ relations across all firms to get $nXaQ_i^{1/\epsilon} - \epsilon^{-1}XaQ_i^{1/\epsilon} = v$, where $Q^* = \sum Q_i^*$. So we find that the optimal total output for the overall industry is $Q^* = [v/(Xa(n - \epsilon^{-1}))]^{-\epsilon}$.

Substitute the expression for $Q^*$ back into the $MR_i = v_i$ relation and solve to get that firm $i$’s optimal output is $Q_i^* = [v_i - v/(n - \epsilon^{-1})][\epsilon'(Xa)][v/(Xa(n - \epsilon^{-1}))]^{-1 - \epsilon}$. Since $P_i^* = aQ_i^{1/\epsilon} = a[v/(Xa(n - \epsilon^{-1}))]$, $R_i^* = Xa[v/(Xa(n - \epsilon^{-1}))][v_i - v/(n - \epsilon^{-1})][\epsilon'(Xa)][v/(Xa(n - \epsilon^{-1}))]^{-1 - \epsilon}$. Firm $i$’s revenue exposure is $\xi_{R_i} = d(lnR_i)/d(lnX) = \epsilon - \epsilon Xv/v + X[v/(n - \epsilon^{-1}) - v]/[v_i(n - \epsilon^{-1}) - v]$, which is summarized in equations (13) and (14).

Firm $i$’s variable operating cost for the optimal output, $V_i^*$, is equal to $v_iQ_i^*$, which is equal to $v_i[v_i - v/(n - \epsilon^{-1})][\epsilon'(Xa)][v/(Xa(n - \epsilon^{-1}))]^{-1 - \epsilon}$. Firm $i$’s variable operating cost exposure, $\xi_{V_i} = d(lnV_i)/d(lnX)$, is shown in equation (15).